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Nonlinear Heating of a Magnetoplasma: Temperature of the Ionosphere

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Results of the theoretical study of nonlinear heating of a multicomponent magnetoplasma (of the ionosphere) under the action of an electric field $E = E_0 e^{i\omega t}$ are considered in this paper. All of the kinds of collisions between the constituents of the plasma (electrons, ions, neutral particles) and their velocities are taken into account. In some cases the self-consistent solution of the corresponding systems of equations is studied. Numerical results characterize altitude, frequency, and angle dependencies of the temperatures of the ionosphere's constituents in the altitude range $Z = 100$–1000 km, in the frequency band $F = 1$–10^4 Hz, and angles $\Theta = 0$–90 deg. The temperature of all of the particles increases very quickly with altitude. At $Z \geq 150$ km, the temperatures in the discussed frequency range become larger than the ionization potential, even when the amplitude of the electric field $E_0 \simeq 1$–2 mV/m.

I. Introduction

THE nonlinear heating of the ionosphere, under the action of an outer source of an electric field $E = E_0 e^{i\omega t}$ has been studied by many authors. This problem has a long history. It began with the papers by Bruevich,[1] Tellegen,[2] Bailey and Martin,[3] Försterling,[4] and by many others in 1933–38 after the discovery of the crossmodulation of radio waves of different frequencies, reflected from the ionosphere, due to the nonlinear interaction of these waves. Shortly after, it was clear that the mechanism of this effect is created by the increase of the velocities of the electrons v_e and of the other constituents of the plasma under the influence of the electric field, namely, the velocities of all of the constituent particles of the ionosphere and the collision frequencies ν between them, and their temperatures $T_{e,i,n}$, i.e., the conductivity of the plasma, become functions of the amplitude E_0 and of the angular frequency ω of the electric field E, where $\nu = \nu(E_0,\omega)$, $\nu = \nu_{e,i,n}[E_0,\omega,\nu(E_0,\omega)]$ and $T = T_{e,i,n}[E_0,\nu(E_0,\omega)]$, and the indexes e, i, and n denote, respectively, electrons, ions, and neutral particles. Thus, the equations that determine these values and describe their behavior in the magnetoplasma become nonlinear. To learn theoretically about this problem, the self-consistent solution of the adequate systems of equations must be used.

It is self-evident that the heating in a collisional plasma acts in large plasma regions. The influence of the electric field covers regions with linear scales much larger than the mean free paths of the particles. Many nice effects created in these regions were described in dozens of papers and in some monographs. Many references to these works can be found in the comprehensive monograph in this field by Gurevich.[5] Earlier references are cited in Alpert.[6]

However, it is seen from the following that the theory of the heating effects, known from the literature (see, for example, Ref. 5), is working in a limited frequency band, namely, when $\omega^2 \gg \omega_L^2 + \dot{\nu}_e \nu_{in}$ ($\omega_L = 2\pi F_L$, F_L is the lower hybrid frequency), i.e., in the ionosphere at frequencies $F \geq (4$–$5)10^4$ Hz. Additionally, in the earlier studies, the influence of the velocities of the ions and the neutral particles, and also the influence of the collision frequencies ν_{in} between the ions and neutral particles, etc., were not taken into account in detail.

In this work, results of a theoretical study of the heating of a magnetoplasma are given, taking into account all kinds of collision frequencies and also their temperature dependencies, i.e., their dependence on the electric field $E = E_0 e^{i\omega t}$. Some results of numerical calculations of the temperatures T_e, T_i, and T_n of the electrons, ions, and neutral particles are presented here for the ionosphere in altitude, frequency, and angle Θ dependencies in the regions $Z = 100$–1000 km, $F = 1$–10^4 Hz (Θ is the angle between the wave's electric field $\boldsymbol{E_0}$ and the geomagnetic field $\boldsymbol{H_0}$).[7] Some of these calculations are based on the self-consistent solution of two systems of equations in the hydrodynamic (microscopic) approximation. One of these systems [see Eq. (1)] consists of three vector (nine scalar) Lorentz equations of the derivatives of the velocities $\boldsymbol{V_e}$, $\boldsymbol{V_i}$, and $\boldsymbol{V_n}$ of the particles. The second system of three equations [see Eq. (2)] consists of the derivatives of the temperatures dT/dt, dT_i/dt, and dT_n/dt.

The general formulas of the self-consistent solution are very complicated and bulky; they are absolutely immense. Only numerical results should be used to learn the $T_{e,i,n}$ and $\boldsymbol{V_{e,i,n}}$ dependencies in this case. However, in the ν = const approximation, i.e., when $\nu_{ei} = \nu_{ei,0}$, $\nu_{en} = \nu_{en,0}$, $\nu_{in} = \nu_{in,0}$, and the electric field dependence of ν is not taken into account, the analytical formulas obtained are sufficiently transparent. These formulas were used for some calculations. It was important to evaluate the applicability of the formulas obtained for ν = const. For this purpose and also for the full study of this problem, the velocities and temperatures are calculated, in some cases, both with the full systems of equations (1–3) and with the shortened systems of Eqs. (1) and (2). In the last case, the neutral particles temperature $T_n = T_{n,0}$ = const.

The dependencies of the temperatures on time, namely, the process of their transition to the stationary state, is illustrated by some examples. The role of the neutral particles is important in this process. The system of equations of the temperatures $(dT_n/dt)_{e,i,n}$ does not have a stationary solution. Mathematically it is clear: the determinant of this system $\Delta = 0$. Therefore T_n increases in time. The temperature of the neutral particles even becomes a source of the heating of the electrons and ions. From the very beginning of the action of the electric field, the source of the heating of the neutral particles is the energy transferred to them by the collisions with the electrons and ions. The values $(T_e - T_n)$ and $(T_i - T_n)$ are positive. However, increasing in time, T_n becomes equal to T_i. From that moment, the degree of the growth of the temperatures becomes large and larger, and the temperature of the electrons

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Table 1 Parameters of the ionsophere

Z, km	100	120	150	200	300	400	500	800	1000
N_n, cm^{-3}	3.7×10^{13}	3×10^{12}	6×10^{11}	5×10^{10}	3×10^9	5×10^8	5×10^7	2×10^6	4×10^5
$N_e = N_i$	10^5	1.2×10^5	1.5×10^5	5×10^5	1.8×10^6	1.5×10^6	10^6	4×10^5	2.3×10^5
$n = N_e/N_n$	$\sim 3 \times 10^{-9}$	4×10^{-8}	2.5×10^{-7}	10^{-5}	6×10^{-4}	3×10^{-3}	2×10^{-2}	2×10^{-1}	5.7×10^{-1}
M^+/M^+_{H1}	28	25	20	12	5	3	2	1.2	1
$(\mu = m/M)10^4$	1.94×10^{-1}	2.36×10^{-1}	2.72×10^{-1}	3.4×10^{-1}	5.4×10^{-1}	1.81	2.72	3.62	5.44
$\sqrt{\mu} \times 10^3$	4.40	4.85	5.21	5.83	7.35	13.4	16.5	19.0	23.3
T, K	220	400	600	1000	1500	~ 1800	~ 1800	~ 1800	2000
$v_e \times 10^{-7}$, cm/s	0.3	0.46	0.63	1.1	2.0	2.8	3.5	4.5	5.2
ν_{en}, s^{-1}	2×10^5	2×10^4	3×10^3	3×10^2	3×10^1	6.0	0.7	10^{-1}	8×10^{-3}
ν_{in}, s^{-1}	8.8×10^2	9.7×10^1	15.6	1.75	0.22	8×10^{-2}	1.15×10^{-2}	1.9×10^{-3}	1.8×10^{-4}
ν_{ei}, s^{-1}	10^3	9×10^2	6×10^2	2×10^3	3×10^3	1.5×10^3	3×10^2	1.5×10^2	70
$(\nu_e \times \nu_{in})$	1.76×10^8	1.1×10^6	5.6×10^4	4.02×10^3	6.67×10^2	1.2×10^2	3.45	2.8×10^{-1}	1.3×10^{-2}
$(\omega_0 \times 10^{-6})$, Hz	17.84	19.54	21.85	39.89	75.69	69.09	56.4	35.6	27.1
$(\Omega_0 \times 10^{-3})$, Hz	78.70	91.20	114	268	790	931	930	758	637
$(\omega_B \times 10^{-6})$, Hz	8.40	8.29	8.20	8.02	7.66	7.31	7.02	6.22	5.68
$(\Omega_B \times 10^{-3})$, Hz	0.163	0.180	0.223	0.272	0.414	1.33	1.91	2.25	3.09
$(\omega_L \times 10^{-4})$, Hz	3.70	3.86	4.28	5.40	7.93	9.86	11.58	13.24	13.25
n_A	490	500	517	859	1326	698	486	289	153
$V_A \times 10^{-7}$, cm/s	6.1	6.0	5.8	3.5	2.3	4.3	6.2	10.3	19.6

comes larger and larger, and the temperature of the electrons becomes larger than it would be without the influence of the neutral particles. The process as a whole does not have a stationary solution. To stop the growth of the temperatures, in addition to collisions, other sources of loss of the energy accompanying the heating of the magnetoplasma must be taken into account (see Sec. II.D).

II. Nonlinear Heating of a Magnetoplasma

The theoretical results of the study of the heating of a magnetoplasma under the action of the electric field $E_0 e^{i\omega t}$ in the microscopic (hydrodynamic) approximation are given in this section. Some results of numerical calculations, mainly of the electron temperature at different altitudes Z of the ionosphere and different angular frequencies $\omega = 2\pi F$, are given in Sec. III. It was noted earlier that the microscopic theory has difficulties from the very beginning because the appropriate system of equations does not have a stationary solution. However, this point becomes crucial at high altitudes of the ionosphere ($Z >, \gg 300$ km) and, especially, in the magnetosphere and in a full ionized plasma, where the collision frequency between the ions and neutral particles $\nu_{en} \sim, = 0$. In this case, the theory must be improved by introducing thermal losses in the adequate equations. Some of these effects are discussed in Sec. II.B.

A. Statement of the Problem: Microscopic Theory

Let us consider a magnetoplasma under the action of an alternative electric field $E = E_0 e^{i\omega t}$, characterized at the starting point of this process $t = 0$ and $E_0 = 0$ by the following:

It is an isothermal plasma, $T_{e0} = T_{i0} = T_{n0}$, and the temperatures of all of the particles are equal.

It is a quasineutral plasma, $N_{e0} = N_{i0}$, and the densities of the electrons and ions are equal.

It uses one kind of neutral particles N_n. The effective mass of different kinds of neutral particles and ions is equal to $M_n = M_i = (\Sigma N_s M_s/N_n)$, where $N_n = \Sigma N_s$, and N_s and M_s are their densities and masses.

Then the temperatures and velocities of the electrons, ions, and neutral particles under the influence of $E_0 e^{i\omega t}$ are described by the following systems of equations:

$$\frac{\mathrm{d}V_e}{\mathrm{d}t} = \frac{eE}{m} - \frac{e}{mc}(V_e \times B) - \nu_{ei}(V_e - V_i) - \nu_{en}(V_e - V_n)$$

$$\frac{\mathrm{d}V_i}{\mathrm{d}t} = \frac{eE}{M} + \frac{e}{Mc}(V_i \times B) + \frac{m}{M}\nu_{ei}(V_e - V_i) - \nu_{in}(V_i - V_n) \quad (1)$$

$$MN_n \frac{\mathrm{d}V_n}{\mathrm{d}t} = mN_e \nu_{en}(V_e - V_n) + MN_i \nu_{in}(V_i - V_n)$$

and

$$\frac{\mathrm{d}T_e}{\mathrm{d}t} = -\frac{2}{3} eV_e \cdot E - \delta_{ei}\nu_{ei}(T_e - T_i) - \delta_{en}\nu_{en}(T_e - T_{n0})$$

$$\frac{\mathrm{d}T_i}{\mathrm{d}t} = -\frac{2}{3} eV_i \cdot E + \delta_{ei}\nu_{ei}(T_e - T_i) - \delta_{in}\nu_{in}(T_i - T_{n0}) \quad (2)$$

$$N_n \frac{\mathrm{d}T_n}{\mathrm{d}t} = N_e \delta_{en}\nu_{en}(T_e - T_{n0}) + N_i \nu_{in}\delta_{in}(T_i - T_{n0})$$

where δ_{en}, δ_{ei}, and δ_{in} are, respectively, the energy lost by the electrons due to their chaotic collisions with the neutral particles and ions and the energy lost by the ions due to their collisions with the neutral particles. In the calculation, the values $\delta_{en} \simeq \delta_{ei} \simeq 2 \times 10^{-3}$ and $\delta_{in} \simeq 1$ were used.[5]

In addition to Eqs. (1) and (2), the following formulas of the collision frequencies must be used:

$$\nu_{ei}(E_0,\omega) = \nu_{ei,0}\left[\frac{T_{e0}}{T_e(E)}\right]^{3/2} \cdot \frac{\ln\{0.37[T_e(E)/e^2 N_e^{1/3}]\}}{\ln[0.37(T_{e0}/e^2 N_e^{1/3})]}$$

$$\nu_{en}(E_0,\omega) = \nu_{en,0}\left[\frac{T_e(E)}{T_{e,0}}\right]^{1/2} \quad (3)$$

$$\nu_{in}(E_0,\omega) = \nu_{in,0}\left[\frac{T_i(E)}{T_{i,0}}\right]^{1/2}$$

where $\nu_{ei,0}$, $\nu_{en,0}$ and $\nu_{in,0}$ are the initial values of the collision frequencies when $E_0 = 0$, and the members $\ln[\ldots]$ are the so-called Coulomb logarithms.

Thus the systems of equations are interconnected and a self-consistent solution of Eqs. (1) and (2) should be used to study in sufficient completeness the problem considered here. Namely, this general statement is the crucial point, both for a full and correct understanding of the process of heating of a magnetoplasma in the frames of the microscopic (hydrodynamic) approximation and also for a correct quantitative estimation of the temperatures.

B. Velocities $V_{e,i,n}$: Approximation ν = const

By solution of the system of Eqs. (1), the formulas of the velocities of the particles are transparent when the dependence of the collision frequencies on the electric field is not taken into account, it is the approximation ν = const. In this case the components of the electron velocities are the following:

$$V_{ex} = \frac{-e\tilde{\omega}_H}{m\omega} E_y \cdot \left\{ \frac{[1 - \tilde{\nu}_{in}^2 - \tilde{\Omega}_H^2 + \mu\tilde{\nu}_{ei}(\tilde{\nu}_{ei} - \tilde{\nu}_{en})] - i(\mu\tilde{\nu}_{ei} + \tilde{\nu}_{in})}{[(1 - \tilde{\nu}_e\tilde{\nu}_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e]^2 - \tilde{\omega}_B^2(1 + i\tilde{\nu}_{in})^2} \right\}$$

$$V_{ey} = \frac{-e}{m\omega} E_y \cdot \left[\frac{i\big((1 + i\tilde{\nu}_{in})[(1 - \tilde{\nu}_e\tilde{\nu}_{in}) + i\tilde{\nu}_e] - \tilde{\Omega}_H^2\{1 + (\tilde{\nu}_{en} - n\tilde{\nu}_{in})(\nu_{in}/\omega + in\nu_{in})]\}\big)}{[(1 - \tilde{\nu}_e\tilde{\nu}_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e]^2 - \tilde{\omega}_H^2(1 + i\tilde{\nu}_{in})^2} \right] \tag{4}$$

$$V_{ez} = \frac{-e}{m\omega} E_z \cdot \left[\frac{i(1 + i\tilde{\nu}_{in})}{(1 - \tilde{\nu}_e\tilde{\nu}_{in}) + i\tilde{\nu}_e} \right]$$

The components of the ion velocity are

$$V_{ix} = \frac{-e\tilde{\Omega}_H}{m\omega} E_y \cdot \left\{ \frac{\mu[1 + i(\mu\tilde{\nu}_{ei} + \tilde{\nu}_{in})]^2 - \tilde{\omega}_L^2 + [\tilde{\nu}_{ei} + in\tilde{\nu}_{en}(\nu_{en}/\omega + in\nu_{in})](2 + i\tilde{\nu}_{in})}{[(1 - \tilde{\nu}_e\nu_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e]^2 - \tilde{\omega}_H^2(1 + i\tilde{\nu}_{in})^2} \right\}$$

$$V_{iy} = \frac{e\mu}{m\omega} E_y \cdot \left(\frac{i\{1 + i[\tilde{\nu}_{en} - n\tilde{\nu}_{en}(\nu_{en}/\omega + in\nu_{in})]\}[(1 - \tilde{\nu}_e\tilde{\nu}_{in}) + i\tilde{\nu}_e]}{[(1 - \tilde{\nu}_e\tilde{\nu}_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e]^2 - \tilde{\omega}_H^2(1 + i\tilde{\nu}_{in})^2} - \Phi_i(\ldots) \right)$$

$$\Phi_i(\ldots) = \left(\frac{i\tilde{\omega}_H^2\{1 + i[\tilde{\nu}_{in} - n\tilde{\nu}_{in}(\nu_{in}/\omega + in\nu_{in})]\}}{[(1 - \tilde{\nu}_e\tilde{\nu}_{in} - \tilde{\omega}_L^2) + i\tilde{\nu}_e]^2 - \tilde{\omega}_H^2(1 + i\tilde{\nu}_{in})^2} \right) \tag{5}$$

$$V_{iz} = \frac{e\mu}{m\omega} E_z \cdot \left(\frac{i\{1 + i[\tilde{\nu}_{en} - n\tilde{\nu}_{en}(\nu_{en}/\omega + in\nu_{in})]\}}{(1 - \tilde{\nu}_e\tilde{\nu}_{in} + i\tilde{\nu}_e} \right)$$

and the velocity of the neutral particles V_n, estimated by the velocities of the electrons and ions V_e and V_i, is equal to

$$V_n = \frac{[V_e + (\tilde{\nu}_{in}/\mu\tilde{\nu}_{en})V_i]}{[1 + (\tilde{\nu}_{in}/\mu\tilde{\nu}_{en}) - (i/n\mu\tilde{\nu}_{en})]} \tag{6}$$

Equations (4-6) are calculated in the coordinate system (x,y,z) where the magnetic field $H_0 = H_0(0,0,H_z)$, i.e., it is parallel to the axis z, and the electric field E_0 lies in the plane (yz), i.e., $E_0 = E_0(0,E_y,E_z)$, $(0, E_0 \sin\Theta, E_0\cos\Theta)$. Additionally, in Eqs. (4-6) the following dimensionless values are used:

$$\tilde{\nu}_{ei} = \frac{\nu_{ei}}{\omega}, \quad \tilde{\nu}_{en} = \frac{\nu_{en}}{\omega}, \quad \tilde{\nu}_{in} = \frac{\nu_{in}}{\omega}$$
$$\tilde{\nu}_e = \frac{\nu_{ei} + \nu_{en}}{\omega}, \quad \tilde{\nu}_e\tilde{\nu}_{in} = \frac{\nu_e\nu_{in}}{\omega^2} \tag{7}$$
$$\tilde{\Omega}_h = \frac{\Omega_H}{\omega}, \quad \tilde{\omega}_H = \frac{\omega_H}{\omega}, \quad \tilde{\omega}_L = \frac{\omega_L}{\omega}$$

where Ω_H, $\omega_L \simeq (\Omega_H\omega_H)^{1/2}$, and ω_H are, respectively, the angular ion, lower-hybrid, and electron gyrofrequencies, $\mu = (m/M)$ and $n = (N_e/N_n) = (N_i/N_n)$.

C. Temperatures $T_e(E)$ and $T_i(E)$ of the Electrons and Ions

By calculating the temperatures, it is convenient and physically more acceptable to use the dimensionaless values ω/ν instead of ν/ω because $\nu_e = \nu_{ei} + \nu_{en}$ is never equal to zero. Then, for example, the components of the velocities $V_{e,z}$ and $V_{i,z}$ (they are the same as in an isotropic plasma) are equal to

$$V_e = V_{ez} = -\frac{eE_z}{m\nu_e} \cdot \frac{[1 - i(\omega/\nu_{in})][1 - (\omega^2/\nu_e\nu_{in}) + i(\omega/\nu_{in})]}{[1 - (\omega^2/\nu_e\nu_{in})]^2 + (\omega^2/\nu_{in}^2)}$$

$$V_i = V_{iz} = -\frac{eE_z}{m\nu_e} \cdot \frac{\nu_{en}}{\nu_{in}} \cdot \frac{[1 - (n\nu_{en}/\omega + in\nu_{in}) - i(\omega/\nu_{en})][1 - (\omega^2/\nu_e\nu_{in}) + i(\omega/\nu_{in})]}{[1 - (\omega^2/\nu_e\nu_{in})]^2 + (\omega^2/\nu_{in}^2)} \tag{8}$$

and their real parts are

$$\mathrm{Re}(V_e) = \frac{-eE_0}{m} \cdot \frac{\nu_e\omega^2 - \nu_{in}(\omega^2 - \nu_e\nu_{in})}{(\omega^2 - \nu_e\nu_{in})^2 + \nu_e^2\omega^2}$$

$$\mathrm{Re}(V_i) = \frac{-eE_0}{M} \cdot \frac{\nu_e\omega^2 - \nu_{en}(\omega^2 - \nu_e\nu_{in})}{(\omega^2 - \nu_e\nu_{in})^2 + \nu_e^2\omega^2} \tag{9}$$

Let us note here that by approximation used in the earlier studies[5] the following equations of the velocity V_e of electrons

$$V_e = -\frac{e}{m}\frac{e^{i\omega t}}{\omega_B^2 + (\nu_e - i\omega)^2} \cdot \left[E_0(\nu_e - i\omega) + \frac{\omega_H^2(E_0H_0)}{H_0^2(\nu_e - i\omega)} H - \frac{\omega_H(E_0H_0)}{H_0} \right] \tag{10}$$

and of $\mathrm{Re}(V_e)$, when $H_0 = 0$,

$$\mathrm{Re}(V_e) = \frac{-eE_0}{m} \cdot \frac{\nu_e}{\omega^2 + \nu_e^2} \tag{11}$$

were used. By comparing Eqs. (10) and (11) with Eqs. (4) and (9), it follows that the equations of V_e and certainly also of the temperatures, used in these studies, are effective only when

$$\omega \gg \nu_{in}, \qquad \omega^2 \gg \nu_e\nu_{in}, \qquad \text{and} \qquad \omega^2 \gg \omega_L^2 + \nu_e\nu_{in} \tag{12}$$

Transparent, but still rather complicated, formulas of the temperatures of the electrons and ions $T_e(E_0,\omega)$ and $T_i(E_0,\omega)$ can be obtained only by the approximation ν = const from the self-consistent solution of the system of Eqs. (1) and the shortened system of Eqs. (2), namely, when in the third equation of Eq. (2) $(\mathrm{d}T_n/\mathrm{d}t) = 0$. These formulas are as follows:

$$\frac{T_e}{T_{n0}} = 1 + \frac{e^2E_0^2}{3m\delta_e\nu_e^2T_{n0}} \cdot \Phi(\ldots) \tag{13}$$

where

$$\Phi(\ldots) = \{A_1(\nu)[\cos^2\Theta \mathrm{Re}(V_{ez}) + \sin^2\Theta \mathrm{Re}(V_{ey}]\} + \{A_2(\nu)[\cos 2\omega t F_{e,1}(\nu,\omega) + \sin 2\omega t F_{e,2}(\nu,\omega)]\} \quad (14)$$

In formula (14), the members proportional to $(m/M) \cdot [\mathrm{Re}(V_{ey},z)]$, and $m/M \cdot \mathrm{Im}(V_{ez})$ are omitted and the following notations are used:

$$A_1(\nu) = \frac{\nu_e + (\nu_{in}/\delta_e)[1 + (\nu_{en}/\nu_{ei})]}{\nu_{en} + (\nu_{in}/\delta_e)[1 + (\nu_{en}/\nu_{ei})]}$$

$$A_2(\nu) = \{[\nu_{en} + (\nu_{in}/\delta_e)]^2 + 4\nu_{ei}^2\}^{-1/2} \quad (15)$$

$$F_{e1}(\nu,\omega) = \frac{A_{3,1}B_1 - 2\omega B_2}{A_{3,1}^2 + 4\omega^2} - \frac{A_{3,2}B_1 - 2\omega B_2}{A_{3,2}^2 + 4\omega^2}$$

$$F_{e2}(\nu,\omega) = \frac{A_{3,1}B_2 - 2\omega B_1}{A_{3,1}^2 + 4\omega^2} + \frac{A_{3,2}B_2 - 2\omega B_1}{A_{3,2}^2 + 4\omega^2} \quad (16)$$

$$B_1(\nu) = \delta_e \nu_{ei}[1 + (\nu_{in}/\delta_e \nu_{ei})][\cos^2\Theta \mathrm{Re}\{V_{ez}\} + \sin^2\Theta \mathrm{Re}\{V_{ey}\}] + 2\omega[\cos^2\Theta \mathrm{Im}\{V_{ez}\} + \sin^2\Theta \mathrm{Im}\{V_{ey}\}]$$

$$B_2(\nu) = \delta_e \nu_{ei}[1 + (\nu_{in}/\delta_e \nu_{ei})][\cos^2\Theta \mathrm{Im}\{V_{ez}\} + \sin^2\Theta \mathrm{Im}\{V_{ey}\}] - 2\omega[\cos^2\Theta \mathrm{Re}\{V_{ez}\} + \sin^2\Theta \mathrm{Re}\{V_{ez}\}] \quad (17)$$

In addition, the values $\mathrm{Re}\{V_{ey}\}$, $\mathrm{Re}\{V_{ez}\}$, $\mathrm{Im}\{V_{ey}\}$, and $\mathrm{Im}\{V_{ez}\}$, are the real and imaginary parts of the terms in braces of the appropriate components of the velocity V_e [see Eq. (4)].

It is seen from Eqs. (13) and (14) that the temperature T_e has two members. One of them estimates the stationary ($t \to \infty$) temperature, not depending on time. The establishment of the stationary temperature is illustrated next by some results of calculations. The second member of Eq. (14) describes the harmonic periodical variation of the temperature; it is changing in time by the double frequency $2\pi f$ of the electric field $E \sim e^{i2\pi f t}$. In the following, we discuss numerical results only of the stationary temperature.

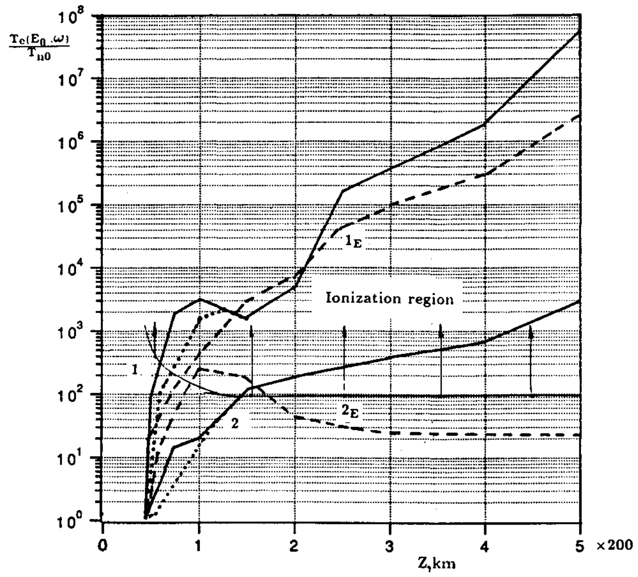

Fig. 1 Altitude dependencies of the ratios $T_e(E_0,\omega)/T_{n0}$ and $T_i(E_0,\omega)/T_{n0}$, $\Theta = 0$: approximation ν = const; $F = 1$ Hz→1, $F = 10^3$ Hz→2; approximation $\nu = \nu(E_0,\omega)$; $F = 1$ Hz→1_E, $F = 10^3$ Hz→2_E [$T_i(E_0,\omega)/T_{n0}$] is given by dots. The region of ionization is marked by a thin line and vertical arrow.

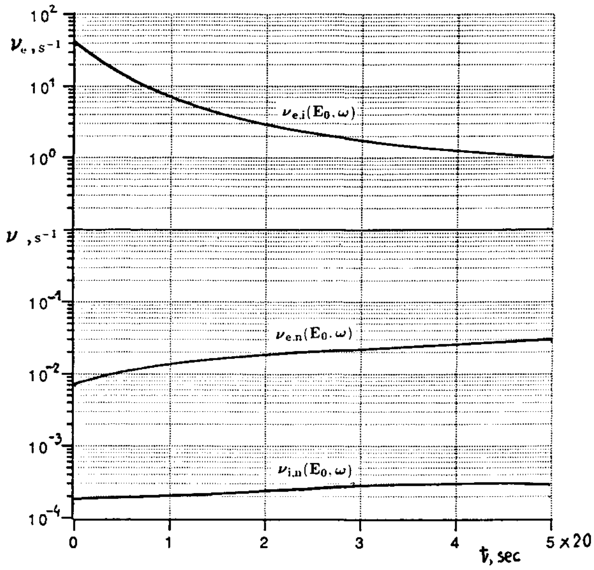

Fig. 2 Establishment of the collision frequencies $\nu(E_0,\omega)$ in time t, $Z = 10^3$ km, $F = 1$ Hz.

It is useful to introduce a characteristic field E_h,[5] which can be, in its own way, a measure of the intensity E_h^2 of the field needed for a remarkable heating of the magnetoplasma. When $\Theta = 0$, i.e., in an isotropic plasma, from Eqs. (13) and (14) it follows that

$$E_h^2 = \frac{3m\delta_e \nu_e^2 T_{n0}}{e^2} \cdot \frac{A(\nu,\omega)}{A_1(\nu)} \quad (18)$$

and the stationary temperature of the electrons is determined by

$$\left[\frac{T_e(E_0,\omega)}{T_{n0}}\right]_{\mathrm{stat}} = 1 + \frac{E_0^2}{E_h^2} \quad (19)$$

The stationary ($t \to \infty$) temperature of the ions is determined by the following equation:

$$\frac{T_i(E_0,\omega)}{T_{n0}} = 1 + \frac{E_0^2}{E_h^2}\left[1 + \frac{1}{\delta_e}\frac{\nu_{in}}{\nu_e}\left(1 + \frac{\nu_{en}}{\nu_{ei}}\right)\right]^{-1} \quad (20)$$

where $A_1(\nu)$ is given by Eq. (15) and

$$A(\nu,\omega) = \frac{[1 - (\omega^2/\nu_e \nu_{in})]^2 + (\omega^2/\nu_{in}^2)}{[1 - (\omega^2/\nu_e \nu_{in})] + (\omega^2/\nu_{in}^2)} = [\nu_e \mathrm{Re}\{V_{ez}\}]^{-1} \quad (21)$$

Certainly we can see by comparing Eqs. (13) and (20) that the temperature $T_i(E_0,\omega)$ of the ions is lower than the temperature $T_e(E_0,\omega)$ of the electrons. However, in the ionosphere this is remarkable only at the low altitudes of the ionosphere $Z \leq 100$–200 km. At $Z \leq 200$ km, $T_i(E_0,\omega) \simeq T_e(E_0,\omega)$.

The time-dependent part of the ion temperature is determined by the same formulas as for the electrons [see Eqs. (13) and (14)], by replacing in the functions $F(\ldots)$ and $B(\ldots)$ [see Eqs. (16) and (17)] the values V_{ey} and V_{ez} of the velocity components of electrons by the values V_{iy} and V_{iz} of the ions. The formulas (4–6) given earlier were used in some numerical calculations of the temperatures with the ν = const approximation (see Sec. III). Because of the diverse character of the problem studied here, let us briefly emphasize again some of its aspects.

D. Some Peculiarities of the Problem

It is obvious that in the frame of the microscopic (hydrodynamic) approximation used in this study, the magnetoplasma should be a collisional medium. However, if it would be, for example, a full ionized magnetoplasma, i.e., if it is character-

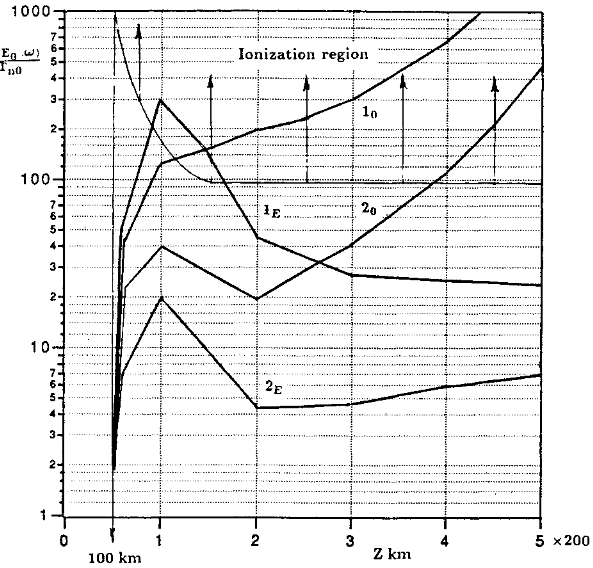

Fig. 3 Same as on Fig. 1 for $F = 10^3$ Hz: approximation $\nu =$ const: $1_0 \to \Theta = 0$; $2_0 \to \Theta = 75$ deg; approximation $\nu = \nu(E_0,\omega)$: $1_E \to \Theta = 0$; $2_E \to \Theta = 75$ deg.

ized only by the constituent particles $N_e = N_i(N_n = 0)$ and by one kind of collision ν_{ei} between the electrons and ions, then the used equations of the temperature would not have a stationary solution. The temperatures of the electrons would increase continuously all the time of the action of the electric field. It is because the loss of the energy of the electrons accelerated by the electric field is equal to the energy transmitted by them to the ions. Mathematically it means that the determinant Δ_2 of the first two equations of the system (2) becomes equal to zero. But this should happen, for example, when we consider these processes in the magnetoplasma at far distances from the Earth where $\nu_{en} \to 0$.

Within the limits of this study, the problem of the absence of a stationary solution of the three equations of Eq. (2) exists from the very beginning if the equation of the derivative of the temperature (dT_n/dt) of the neutral particles is taken into account. Mathematically it is because the determinant of the system in Eq. (2) $\Delta_3 = 0$. The temperature T_n of the neutral particles is increasing continuously under the action of the electric field and can even become a source of heating of the electrons and ions. The crucial region of the ionosphere, where this process becomes highly noticeable, is at the altitudes $Z > 300$ km. The values T_e and T_i become larger in this region than they would be by the solution of the shortened system of Eqs. (2).

Thus it was important to learn the applicability of the microscopic approximation used in many studies and here by considering the behavior of all of the temperatures T_e, T_i, and T_n by the two approximations: $\nu =$ const and $\nu(E)$. Especially it was important to study the role of the neutral particles. The appropriate results of this study are given in the next section. Here let us only shortly note how the continuous growth of the temperatures of the plasma can be stopped.

The heating of the magnetoplasma is accompanied and regulated, in addition to the collisions ν_{ei}, ν_{ei}, and ν_{in}, by many other processes. Their role should be of different degree. One of the most important of these processes is the ionization of the neutral particles by the accelerated electrons under the action of the electric field $E_0 e^{i\omega t}$.

The velocity of the electrons becomes in the ionosphere, depending on the frequency ω and altitude Z, larger than the ionization potential E_i of the neutral particles. Namely, the thermal, chaotic velocity $v_e(E)$ plays the prime role by this process. It is considerably larger than the directed velocity $V_e(E)$. For instance, in an isotropic plasma

$$v_e(E) = \left[\frac{2\kappa T_e(E_0,\omega)}{m}\right]^{1/2} \sim \frac{\mathrm{Re}[V_e(E)]}{\sqrt{\delta}} \sim 20Re[V_e(E)] \quad (22)$$

[see Eqs. (5) and (9)]. The ionization potential of the atomic hydrogen H_1 particles (they are the main constituent at the high altitudes of the ionosphere) is equal to $E_i = 13.54$ eV. Therefore, the process of the ionization begins when $v_E(E) = v_{e(i)} \geq 2.2 \times 10^8$ cm/s. It becomes sufficiently remarkable that the additional density of the electrons $N_e(E)$ can become even considerably larger than the regular density of the plasma N_e, when the amplitude of the electric field is rather small, even when $E_0 = 1$-2 mV/m. This is seen from the following.

The cross section $\sigma_e(E_i)$ of ionization of the atomic hydrogen H_1 increases rapidly with E_i up to a sharp maximum $\sigma_{e,\max} \simeq (7\text{-}8)10^{-17}$ cm^2 and then it smoothly decreases to $\sigma_{e,\mathrm{stat}} \simeq 2 \times 10^{-17}$ cm^2 (see Ref. 8), i.e., at the altitude, for example, $Z = 300$ km, where the density of H_1 $N_n \simeq 3 \times 10^9$, the production of the electrons

$$I_e = \sigma_e \cdot N_n \cdot v_e(E) \geq (10\text{-}50)\ \mathrm{s}^{-1} \quad (23)$$

The additional ionization is described by the recombination equation

$$\frac{dN}{dt} = I_e N - \alpha_e N^2 \quad (24)$$

It follows from Eqs. (23) and (24) that, increasing in time, the electron concentration reaches a maximum of $(dN/dt) = 0$ equal to $N \simeq 10^9$, even if the coefficient of recombination $\alpha_e \sim 10^{-8}$, cm$^3 \cdot$ s^{-1}. However, in this region of the ionosphere, $\alpha_e <, \ll 10^{-9}$. Thus, under the action of the electric field, the surrounding plasma can become fully ionized, i.e., $N_e \simeq 10^9$ cm^{-3}. As a result, the collision frequency between the electrons and ions $\nu_{ei}(E) \propto N(e)/T_e(E)^{3/2}$ becomes very large [see Eq. (3)]. Besides, the growth of T_e should be stopped very quickly because the electron concentration increases very quickly. Indeed, in the point of inflection of the time dependence of the $N(E,t)$, where $(d^2N/dt^2) = 0$, $N = (I_e/2\alpha)$. Thus, at that point $(dN/dt) = (I_e^2/4\alpha_e)$ is very large and the tangential of $N(E,t)$ is almost vertical.

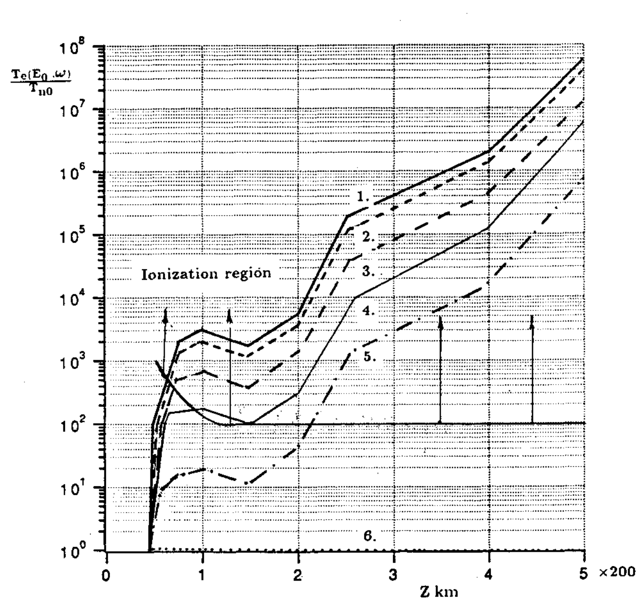

Fig. 4 Same as on Fig. 1: approximation $\nu =$ const, $F = 1$ Hz; 1, 2, 3, 4, 5, and $6 \to \Theta = 0$, 30, 60, 75, 85, 90 deg between the vectors E_p and H_0.

Another loss of energy that can stop the growth of the temperature of the magnetoplasma is, for example, the thermal emission of the constituent particles of the plasma, i.e., the volume Stefan-Boltzmann emission of the plasma. The density of this emission is approximately equal to

$$I_p(\omega) = \frac{\kappa T \omega^2}{4\pi^3 c^3} \cdot n_{gr}(\omega) n^2(\omega), \qquad \text{erg} \cdot \text{s} \cdot \text{cm}^3 \tag{25}$$

In Eq. (25), $n(\omega)$ is a coefficient of refraction of the plasma and $n_{gr} = n + \omega(dn/d\omega)$ is the so-called group velocity coefficient of refraction. The heating kinetic losses of the plasma should be also considered, i.e., the electron-electron and ion-ion collisions ν_{ee} and ν_{ii} and possibly other kinds of kinetic thermal losses that, in a sense, are similar to the Landau damping of electromagnetic waves in a plasma.

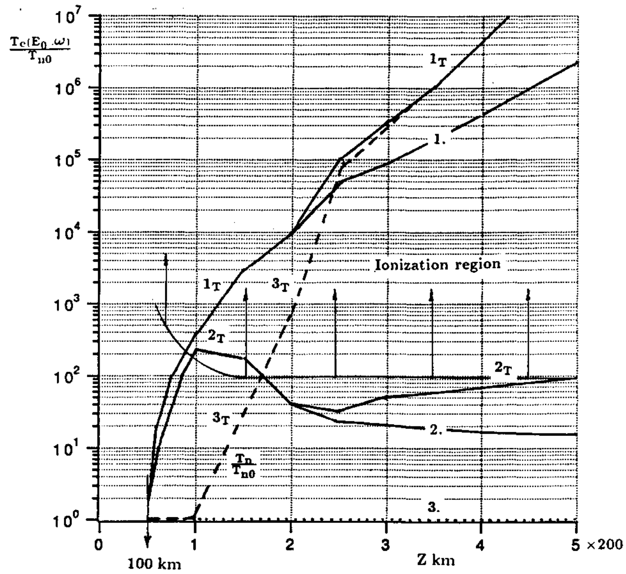

Fig. 5 Altitude dependencies of the ratios of the temperatures: approximation $\nu = \nu(E_0,\omega)$, $\Theta = 0$; $T_e(E_0,\omega)/T_{n0} : F = 1$ Hz, $1 \to (dT_n/dt) = 0$, $1_T \to (dT_n/dt) \neq 0$; $T_n(E_0,\omega)/T_{n0} : F = 1$ Hz, $3 \to (dT_n/dt) = 0$, $3_T \to (dT_n/dt) \neq 0$.

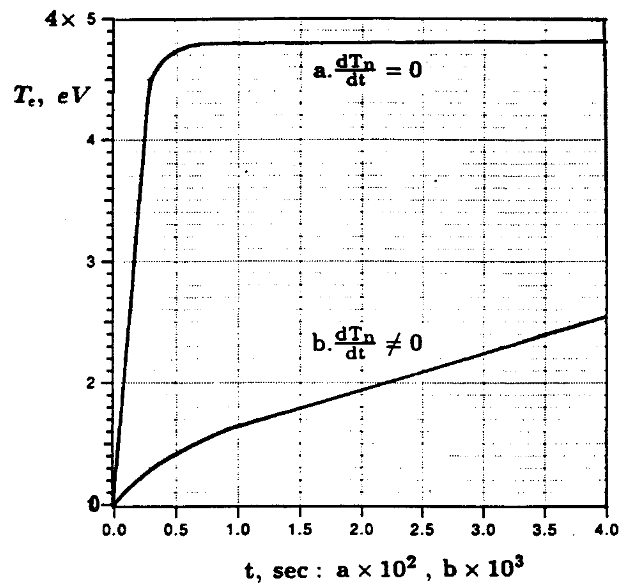

Fig. 6 The establishment of the temperature $T_e(E_0,\omega)$ in time; $F = 1$ Hz, $\Theta = 0$, $Z = 200$ km; approximation $\nu = \nu(E_0,\omega)$: a)$\to (dT_n/dt) = 0$; b)$\to (dT_n/dt) \neq 0$.

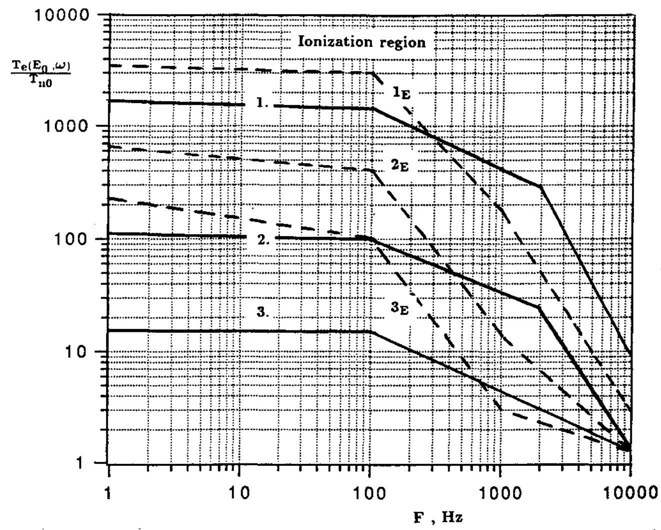

Fig. 7 Frequency dependencies of the ratios $T_e(E_0,\omega)/T_{n0}$, $Z = 300$ km; approximation ν = const : $1 \to \Theta = 0$; $2 \to \Theta = 75$ deg, $3 \to \Theta = 85$ deg; approximation $\nu = \nu(E_0,\omega)$: $1_E \to = 0$; $2_E \to \Theta = 75$ deg; $3_E \to \Theta = 85$ deg.

We omit in this paper the discussion in detail of the issue noted earlier and some other processes accompanying the heating of the magnetoplasma by an electric field. This is beyond the frame of this study. Certainly, by taking into account these effects, the discussed problem becomes rather complicated. Additional members (even equations) should be included in the Eqs. (2) and should be used by the general solution of the systems (1) and (2).

III. Numerical Results: Conclusions for $Z = 10^2$-10^3 km and $F = 1$-10^4 Hz

In the following section, we give results of numerical calculations mainly of the temperature of the electrons and, for comparison, a few examples of calculation of the temperatures of the ions and neutral particles. The following model of the ionosphere (see Table 1) and the amplitude of the electric field $E_0 = 1$ mV/m are used by these calculations. The altitude dependencies of the electron and ion densities and of the Lengmuir frequencies $f_0 = \omega_0/2\pi$ and $F_0 = \Omega_0/2\pi$, given in Table 1, correspond to the daytime conditions of the ionosphere. The altitude, frequency, and angle θ dependencies of the temperatures are presented in plots (Figs. 1-7). The frequencies $F = \omega/2\pi = 1$ and 10^3 Hz and the angle between the magnetic and electric field $\Theta = 0$ and the altitude $Z = 300$ km are selected by the calculations to illustrate the most important results of this study.

It is seen later that the growth of the temperature with altitude in the frequency range $F \simeq 1$-100 Hz is very large. It becomes about 10^2-10^3 and many times larger than the initial temperature of the plasma already at $Z \sim 200$-300 km. Therefore, the frequency $F = 1$ Hz is used to illustrate more dramatically the region where it is necessary to stop the temperature growth by including in the theory additional losses of energy (see Sec. II.D). The zone of ionization of the plasma by the accelerated electrons is therefore marked on the plots by a thin line and vertical arrows. The frequency band $F = 1$-10^4 Hz that we have used is much smaller but also becomes somewhat larger than the gyrofrequency $F_H = (\Omega_H/2\pi)$ of the ions in the altitude range discussed. It is also much smaller than the low hybrid frequency $F_L = (\omega_L/2\pi)$. In addition, the value $\nu_e \nu_{in} = (\nu_{ei} + \nu_{in})\nu_{in}$ is much larger than these frequencies at low altitudes and becomes comparable and also much larger than at high altitudes. Therefore, the formulas used in the literature are ineligible for these calculations. The used angle $\Theta = 0$ corresponds to the isotropic plasma and characterizes the behavior of the maximum values of the temperature. However, the evolution of these dependencies on different values of Θ are also given and the influence of the magnetic field H_0 is shown by these data.

The results of the numerical calculations mainly correspond to the approximation $\nu = \text{const}$, i.e., the shortened system of Eq. (2) is used and it is assumed that $(dT_n/dt) = 0$, $T_n = T_{n0}$. However, results of the self-consistent solution of the full systems of Eqs. (1–3) are also given by some examples and the role of the neutral particles is discussed.

Let us now describe in more detail the numerical results by the plots presented here.

A. $(dT_n/dt) = 0$

First of all, let us consider the altitude dependencies of the ratio of the temperatures $T_e(E_{0,\omega})$ and $T_i(E_{0,\omega})$ to the initial temperature of the plasma T_{n0}, shown on Fig. 1 for $F = 1$ and 10^3 Hz, calculated by the shortened system of Eqs. (2) and by the two approximations: $\nu = \text{const}$ and $\nu = \nu(T) = \nu(E_{0,\omega})$. How the collision frequencies $\nu(E,t)$ reach their stationary values at the altitude $Z = 103$ km and $F = 1$ Hz, is also shown for sake of illustration, on Fig. 2.

The following is seen by examining Fig. 1.

1) The temperatures of the electrons and ions increase quickly on both frequencies. They become equal in the altitude region $Z \sim 200$–250 km.

2) The temperatures, when $F = 10^3$ Hz, have a maximum at $Z \sim 200$ km by the approximation $\nu = \nu(E_0,\omega)$. The same happens by approximation $\nu = \text{const}$ when $F = 1$ Hz.

3) At $Z \geq 300$ km, the temperatures are smaller by the approximation $\nu = \nu(E_0,\omega)$ than by the approximation $\nu = \text{const}$.

4) The process of ionization by the accelerated electrons is acting almost in all of the altitude regions when $F = 1$ Hz and does not play a role, when $F = 10^3$ Hz, especially at $Z \geq 300$ km. Thus, the process of the ionization should be stopped when $F = 1$ Hz at the altitude $Z = 120$–150 km.

These properties of the nonlinear heating process of the ionosphere are typical. Namely, the behavior of T_e and T_i for $F = 1$ Hz is typical for the frequency band $F = 1$–102 Hz and at frequencies a little higher. The behavior for $F = 10^3$ Hz is typical for the frequency band $F > 10^2$–10^3 Hz (see Fig. 7). The altitude dependencies of $T_e(E_0,\omega)/T_{n0}$ given on Figs. 3 and 4 illustrate in more detail these characteristics. It is also seen that the temperature diminishes very quickly with the angle when $60 \text{ deg} < \Theta \to 90$ deg.

B. $(dT_n/dt) \neq 0$

Some results of the self-consistent solution of the full systems of Eqs. (1–3) are presented in Fig. 5. Not to overload this figure, only the T_e and T_n dependencies are given in it. The main new important characteristics of these dependencies in addition to those given in the previous section are the following:

1) The temperature T_n of the neutral particles increases very quickly at altitudes $Z > 200$ km. It approaches the temperature T_e of the electrons at $Z > 400$–500 km and becomes close to T_e similarly to the temperature of ions, T_i.

2) At the altitudes $Z > 400$ km, the temperatures T_e are larger on both the frequencies by the approximation $(dT_n/dt) \neq 0$ than by the approximation $(dT_n/dt) = 0$.

3) At least at $Z \geq 300$ km, the heating of the neutral particles becomes a source of the heating of the electrons and ions. The growth of T_n should be stopped, similarly to the case $(dT_n/dt) = 0$ at these altitudes (See item 4 in Sec. III.A).

4) In general, the heating of the neutral particles is an important part in many cases of the process of heating of the plasma.

The establishment of the temperature $T_e(E_0,t)$ to its stationary value is shown on Fig. 6 for $F = 1$ Hz and $\Theta = 0$. This process goes on very slowly when $(dT_n/dt) \neq 0$, compared with the case $(dT_n/dt) = 0$.

C. $T_e(F, \Theta)$

Frequency dependencies of the temperature T_e of the electrons at $Z = 300$ km and different values of the angle Θ, calculated for the both approximations and $E_0 = 1$ mV/m, are given in Fig. 7. In a sense, $Z = 300$ km is a transition region because the heating of the neutral particles is still not too large in this region (see Fig. 5).

The behavior of the frequency dependencies of T_e, are in agreement with those earlier conclusions in Secs. III.A. and III.B. The heating of all kinds of particles is very high in this region. Even by amplitudes of the field $E_0 \sim 10^{-2}$–10^{-1} V/m, the temperatures of the electrons and ions can rapidly reach values $T_e(E_0,\omega) \geq (10$–$10^2)T_{n0}$ in a broad range of angles Θ between the electric field E_0 and the Earth's magnetic field H_0. Temperatures $T_e(E_0,\omega) > 10^2 T_{n0}$ should at least be stopped by the ionization and recombination and other processes accompanying the heating of the plasma by the electric field $E = E_0 e^{i\omega t}$. This does not mean that the temperatures of the electrons and ions cannot become larger than the critical temperature $T_{e(i)}$ of ionization, let us say, of the atomic hydrogen H_1, which is equal to

$$T_{e(i)} = 13.5 \text{ eV} = 1.16 \cdot 10^4 \cdot 13.5 = 1.57 \cdot 10^5, \text{ K} \tag{26}$$

However, to answer this question, theoretical calculation must be done, taking into account losses of energy, in particular, noted in Sec. II.D.

Summary

The theory of nonlinear heating of a collisional magnetoplasma in the microscopic approximation was extended, taking into account the heating of all the kinds of particles, all of the kinds of collisions between them, and also the interconnection of these processes.

The numerical results describe in detail the altitude, frequency, and angle temperature dependencies of all of the constituents of the ionosphere in the frequency range $F = 1$–10^4 Hz and altitude region $Z = 10^2$–10^4 km.

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